Kinetic model of three component, weakly ionized, collisional plasma with a beam of neutral particles

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Kinetic model of three component, weakly ionized, collisional plasma with a beam of neutral particles is developed. New dispersion relations for linear perturbations are derived and analyzed in various limiting cases.

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I. INTRODUCTION

It is well-known that neutral beam injection is one of the fundamental fusion plasma heating methods. In general, a particle accelerator is used to create fast ion beams (the particle energies are on the order of 100 keV); the ion beam is then passed through a neutral gas region, where the ions neutralize via charge-exchange reactions with the neutral gas. The neutralized beam is then injected into a magnetically confined plasma. Of course, the neutral atoms are unaffected (not confined) by the magnetic field, but ionize as they penetrate into the plasma. Then the high-energy ions transfer fraction of their energy to the plasma particles in repeated collisions, and heat the plasma [1–6].

In this paper we develop a kinetic model of three component, weakly ionized, collisional plasma with a beam of neutral particles. We employ a kinetic equation for the charged particles of α sort in the weakly ionized plasma with the Batnagar-Gross-Krook (BGK) model collisional term. Similar model has been developed previously by others [7]. In this book authors do not take into account possibility of existence of regular velocity of the neutral particles [7]. In the light of the possible relevance of our model for the heating of plasma by neutral beam injection, we set out with the aim to generalize results of Ref. [7] by allowing neutral particles to have regular velocity and seek for possible novelties brought about by this effect. Indeed, the dispersion relations for linear perturbations obtained in this paper differ substantially from those of Ref. [7].

In section II we formulate our model and obtain general dispersion relation. In section III we analyze various limiting cases of the dispersion relation and discuss the results.

II. THE MODEL

We start analysis of the dielectric permittivity (DP) of a collosional plasma with weakly ionzed, non-degenerate plasma when the integral of elastic collisions in the kinetic equation for the charged particles can be apporximated by the BGK term, while it is possible to neglect the collisions between the chaged particles themselvs. Analysis of this relatively simple model will be useful for further more complicated case of fully ionized plasma (which, in fact, is more relevant for the fusion plasma). The latter is beyond the scope of present paper and the separate analysis needs to be done.

The kinetic equation for the charged particles of α sort in the weakly ionized plasma with the BGK model collisional term can be written as following [7]:

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}\cdot\frac{\partial f_\alpha}{\partial \vec{r}} + e_\alpha\{\vec{E} + \vec{v}\times\vec{B}\}\frac{\partial f_\alpha}{\partial \vec{p}} = -\nu_{\alpha n}(f_\alpha - N_\alpha \Phi_{\alpha n}). \tag{1}$$

Here, $\nu_{\alpha n}$ denotes collision frequency of charged particles with the neutrals, which in this model is assumed being constant, whereas

$$N_\alpha \equiv \int d\vec{p} f_\alpha,$$

and

$$\Phi_{\alpha n} \equiv \frac{1}{(2\pi m_\alpha T_{\alpha n})^{3/2}} \exp\left[-m_\alpha(\vec{v}-\vec{V}_0)^2/(2T_{\alpha n})\right], \quad T_{\alpha n} \equiv \frac{m_\alpha T_n + M_n T_\alpha}{m_\alpha + M_n}. \tag{2}$$

Index α ($\alpha = e, i$) refers to charged particles (electrons and ions), whereas n — to neutrals. $\vec{V}_0$ denotes regular, uniform velocity of the neutral particles. Finally, T_α is defined by following expression:

$$T_\alpha = \frac{m_\alpha}{2N_\alpha} \int d\vec{p} (\vec{v} - \vec{V}_\alpha)^2 f_\alpha$$

The specific form of the BGK integral used here is derived from its more general form [7]

$$\left(\frac{\partial f_\alpha}{\partial t} \right)^{\alpha\beta}_{BGK} = -\nu_{\alpha\beta} (f_\alpha - N_\alpha \Phi_{\alpha\beta}), \tag{3}$$

where $\nu_{\alpha\beta}$ is some constant which has meaning of effective collision frequency between particles of α and β sort, i.e. it characterizes time of momentum relaxation of α sort particles as a result of their collision with particles of β sort. Function $\Phi_{\alpha\beta}$ is determined by following expression:

$$\Phi_{\alpha\beta} \equiv \frac{1}{(2\pi m_\alpha T_{\alpha\beta})^{3/2}} \exp\left[-m_\alpha (\vec{v} - \vec{V}_\beta)^2 / (2T_{\alpha\beta}) \right], \tag{4}$$

here $V_\beta = (1/N_\beta) \int d\vec{p}\, \vec{v} f_\beta$.

It should be emphasized that the BGK collisonal integral describes accurately collisions only particles of different sort. Therefore, it can be used to describe collisions of charged paricles with the neutrals in weakly ionized plasma, when the scatteing of charged particles on the neutrals is a dominant process. In the case of fully ionized plasma, in spite of its relative simplicity, use of BGK integral is not justified [7].

In what follows, we consider isothermal models of the BGK integral, i.e. we neglect change in temperature of charged particles with chage in their corresponding distribution functions. We ought to mention that the results obtained here will be qualitatively the same for the non-isothermal model of BGK integral. We further assume that the masses and the temperatures of the ions and neutrals do coincide, i.e. $m_i = M_n \equiv M$ and $T_i = T_n$. In this case to the order of $\sim m_e/M$ terms we have $T_{en} = T_e$. Thus, in the Eq.(2), under these simplifying assumptions we can set $T_{\alpha n} = T_\alpha$ and

$$\Phi_{\alpha n} = \frac{1}{(2\pi m_\alpha T_\alpha)^{3/2}} \exp\left[-m_\alpha (\vec{v} - \vec{V}_0)^2 / (2T_\alpha) \right] \tag{5}$$

which, in fact, coincides with the Maxwellian distribution function (with the beam having velocity $\vec{V}_0$) normalized to unity.

In the static equilibrium state, with the external fields absent, Eq.(1) allows for the only solution $f_{0\alpha} = N_{0\alpha}\Phi_{0n}$. In what follows subscript 0 will denote unperturbed and δ perturbation of the physical quantities.

Let us consider small perturbation of the distribution function δf_α which is caused by appearance of small fields $\vec{E}$ and $\vec{B}$. After usual linearization of the Eq.(1) we obtain

$$\frac{\partial \delta f_\alpha}{\partial t} + \vec{v} \cdot \frac{\partial \delta f_\alpha}{\partial \vec{r}} + e_\alpha \vec{E} \cdot \frac{\partial f_{0\alpha}}{\partial \vec{p}} = -\nu_{\alpha n} (\delta f_\alpha - \int d\vec{p} \delta f_\alpha \Phi_{\alpha n}). \tag{6}$$

The solution of the latter equation for the plane monochromatic waves (i.e. $\vec{E}, \delta f_\alpha \sim \exp\left[-i\omega t + i\vec{k} \cdot \vec{r} \right]$) can be written as

$$\delta f_\alpha = i \frac{e_\alpha}{T_\alpha} \frac{f_{0\alpha} \left[\vec{v} \cdot \vec{E} - \vec{V}_0 \cdot \vec{E} \right]}{\omega + i\nu_{\alpha n} - \vec{k} \cdot \vec{v}} + \frac{i\nu_{\alpha n} \eta_\alpha f_{0\alpha}}{\omega + i\nu_{\alpha n} - \vec{k} \cdot \vec{v}}, \tag{7}$$

where $\eta_\alpha = (1/N_{0\alpha}) \int d\vec{p} \delta f_\alpha$, which is perturbation of the particle number density normalized to equilibrium value of the number dinsity. η_α can be calculated either by integration of the Eq.(7) over momentum or by using the continuity equation for the particles of α sort:

$$\eta_\alpha = \frac{\vec{k} \cdot \vec{j}_\alpha}{e_\alpha N_{0\alpha} \omega},$$

$$\vec{j}_\alpha = e_\alpha \int d\vec{p}\, \vec{v} \delta f_\alpha, \tag{8}$$

here, $\vec{j}_\alpha$ denotes charge current of particles of α sort.

It is known that the complex tensor of DP can be written as

$$\varepsilon_{ij}(\omega,\vec{k}) = \delta_{ij} + \frac{i}{\varepsilon_0\omega}\sigma_{ij}(\omega,\vec{k}) \tag{9}$$

where δ_{ij} is usual Kroneker tensor and $\sigma_{ij}(\omega,\vec{k})$ is the conductivity tensor defined by

$$j_i = \sum_\alpha j_{i\alpha} = \sigma_{ij}(\omega,\vec{k})E_j. \tag{10}$$

In general when ε_{ij} tensor is of the type $\varepsilon_{ij} = \delta_{ij} + A_iA_j - A_iB_j$, then defining quantities ε^l and ε^{tr} as

$$\varepsilon^l = \frac{k^ik^j}{k^2}\varepsilon_{ij} = 1 + \frac{(\vec{k}\cdot\vec{A})^2}{k^2} - \frac{(\vec{k}\cdot\vec{A})(\vec{k}\cdot\vec{B})}{k^2} \tag{11}$$

and

$$\varepsilon^{tr} = \frac{1}{2}\left(\delta^{ij} - \frac{k^ik^j}{k^2}\right)\varepsilon_{ij} = 1 + \frac{(\vec{k}\times\vec{A})^2}{2k^2} - \frac{(\vec{k}\times\vec{A})(\vec{k}\times\vec{B})}{2k^2} \tag{12}$$

respectively, we can split tensor from Eq.(9) in the longitudinal and transverse (with respect to wave-vector $\vec{k}$) parts as following:

$$\varepsilon_{ij}(\omega,\vec{k}) = \left(\delta_{ij} - \frac{k_ik_j}{k^2}\right)\varepsilon^{tr}(\omega,k) + \frac{k_ik_j}{k^2}\varepsilon^l(\omega,k) \tag{13}$$

Now, inserting expression for δf_α from the Eq.(7) into Eq.(8) and using Eqs.(10)-(12) we obtain following expessions for ε^l and ε^{tr}:

$$\varepsilon^l = 1 - \sum_\alpha \frac{\omega_{L\alpha}^2}{k^2\omega}\frac{1}{(2\pi)^{3/2}}\frac{1}{V_{T\alpha}^5}\left[\int d\vec{v}\frac{(\vec{k}\cdot\vec{v})^2 e^{-v^2/(2V_{T\alpha}^2)}}{\omega + i\nu_{\alpha n} - \vec{k}\cdot\vec{v}} - \int d\vec{v}\frac{(\vec{k}\cdot\vec{v})(\vec{k}\cdot\vec{V}_0)e^{-v^2/(2V_{T\alpha}^2)}}{\omega + i\nu_{\alpha n} - \vec{k}\cdot\vec{v}}\right]\times$$

$$\left[1 - \frac{i\nu_{\alpha n}k_i}{\omega}\frac{1}{(2\pi)^{3/2}V_{T\alpha}^3}\int d\vec{v}\frac{v_i e^{-v^2/(2V_{T\alpha}^2)}}{\omega + i\nu_{\alpha n} - \vec{k}\cdot\vec{v}}\right]^{-1}, \tag{14}$$

$$\varepsilon^{tr} = 1 - \sum_\alpha \frac{\omega_{L\alpha}^2}{2k^2\omega}\frac{1}{(2\pi)^{3/2}}\frac{1}{V_{T\alpha}^5}\left[\int d\vec{v}\frac{(\vec{k}\times\vec{v})^2 e^{-v^2/(2V_{T\alpha}^2)}}{\omega + i\nu_{\alpha n} - \vec{k}\cdot\vec{v}} - \int d\vec{v}\frac{(\vec{k}\times\vec{v})(\vec{k}\times\vec{V}_0)e^{-v^2/(2V_{T\alpha}^2)}}{\omega + i\nu_{\alpha n} - \vec{k}\cdot\vec{v}}\right]. \tag{15}$$

Here, $\omega_{L\alpha} = \sqrt{(e^2N_\alpha)/(\varepsilon_0 m_\alpha)}$ and $V_{T\alpha} = \sqrt{T_\alpha/m_\alpha}$. The integrals in the Eqs.(14) and (15) may be evaluated by choosing the z-axis along $\vec{k}$. The integration over v_x and v_y is elementary. Whereas, v_z integral may be expressed in terms of a single transcendental function, which called the plasma dispersion function. There are several different definitions of this function used in the literature. We use the one given by Melrose [8]:

$$\bar{\phi}(z) = -\frac{z}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{dte^{-t^2}}{t-z}. \tag{16}$$

Using Eq.(16) and following intermediate results of integration

$$\int d\vec{v}\frac{(\vec{k}\cdot\vec{v})^2 e^{-v^2/(2V_{T\alpha}^2)}}{\omega + i\nu_{\alpha n} - \vec{k}\cdot\vec{v}} = -\sqrt{\pi}z[\bar{\phi}(z) - 1], \tag{17}$$

$$\int d\vec{v}\frac{(\vec{k}\cdot\vec{v})(\vec{k}\cdot\vec{V}_0)e^{-v^2/(2V_{T\alpha}^2)}}{\omega + i\nu_{\alpha n} - \vec{k}\cdot\vec{v}} = (2\pi)^{3/2}V_{T\alpha}^3(\vec{k}\cdot\vec{V}_0)[\bar{\phi}(z) - 1], \tag{18}$$

$$\frac{i\nu_{\alpha n}k_i}{\omega}\frac{1}{(2\pi)^{3/2}V_{T\alpha}^3}\int d\vec{v}\frac{v_i e^{-v^2/(2V_{T\alpha}^2)}}{\omega + i\nu_{\alpha n} - \vec{k}\cdot\vec{v}} = \frac{i\nu_{\alpha n}}{\omega}[\bar{\phi}(z) - 1], \tag{19}$$

$$\int d\vec{v}\frac{(\vec{k}\times\vec{v})^2 e^{-v^2/(2V_{T\alpha}^2)}}{\omega + i\nu_{\alpha n} - \vec{k}\cdot\vec{v}} = \frac{\sqrt{\pi}}{zk}\bar{\phi}(z), \tag{20}$$

$$\int d\vec{v}\frac{(\vec{k}\times\vec{v})(\vec{k}\times\vec{V}_0) e^{-v^2/(2V_{T\alpha}^2)}}{\omega + i\nu_{\alpha n} - \vec{k}\cdot\vec{v}} = 0, \tag{21}$$

where, $z = (\omega + i\nu_{\alpha n})/(\sqrt{2}kV_{T\alpha})$, we obtain

$$\varepsilon^l = 1 + \sum_\alpha \frac{\omega_{L\alpha}^2}{k^2V_{T\alpha}^2}\frac{[1-\bar{\phi}(z)][1-(\vec{k}\cdot\vec{V}_0)/(\omega+i\nu_{\alpha n})]}{1-[(i\nu_{\alpha n})/(\omega+i\nu_{\alpha n})]\bar{\phi}(z)}, \tag{22}$$

$$\varepsilon^{tr} = 1 - \sum_\alpha \frac{\omega_{L\alpha}^2}{\omega(\omega+i\nu_{\alpha n})}\bar{\phi}(z). \tag{23}$$

Note, that conventinal kinetic model of three component, weakly ionized, collisional plasma [7] is significantly modified by taking into account possible existence of a beam of neutral particles. Namely, the expression for the ε^l is modified by additional factor $[1-(\vec{k}\cdot\vec{V}_0)/(\omega+i\nu_{\alpha n})]$. While the form of the ε^{tr} is not changed by the presence of the beem.

III. DISCUSSION

Let us start analysis of the obtained results from longitudinal waves as we have seen that transverse waves do not incur any modification by the presence of the beem of neutral particles. The dispersion relation for the longitudinal waves reads as following:

$$\varepsilon^l = 1 + \sum_\alpha \frac{\omega_{L\alpha}^2}{k^2V_{T\alpha}^2}\frac{[1-\bar{\phi}(z)][1-(\vec{k}\cdot\vec{V}_0)/(\omega+i\nu_{\alpha n})]}{1-[(i\nu_{\alpha n})/(\omega+i\nu_{\alpha n})]\bar{\phi}(z)} = 0 \tag{24}$$

The latter equation is a transcendental one, thus, in general case, it has many complex solutions $\omega(k)$. Let us consider the most interesting ones which correspond to weakly damped oscillations.

Let us consider, first, high frequency waves, i.e. when $\omega \gg kv_{T\alpha}, \nu_{\alpha n}$. Using asymptotic expansion for $\bar{\phi}(z)$ [8]

$$\bar{\phi}(z) = 1 + \frac{1}{2z^2} + \frac{3}{4z^4} + ... - i\sqrt{\pi}ze^{-z^2}, \quad \text{when} \quad |z| \gg 1 \tag{25}$$

we obtain following dispersion relation for the weakly damped waves (Re $\omega \gg$ Im ω)

$$\varepsilon^l = 1 - \left[\frac{\omega_{Le}^2}{\omega^2}\left(1+\frac{3k^2V_{Te}^2}{\omega^2}\right) - i\left\{\sqrt{\frac{\pi}{2}}\frac{\omega\omega_{Le}^2}{k^3V_{Te}^3}\exp\left[-\frac{\omega^2}{2k^2V_{Te}^2}\right] + \frac{\omega_{Le}^2\nu_{en}}{\omega^3}\right\}\right]\left[1-\frac{\vec{k}\cdot\vec{V}_0}{\omega}\right] = 0. \tag{26}$$

Here, we neglect the contribution from ions, because it is significant when $T_i \geq T_e(M/m_e)^2$, i.e. when the temperature of ions is greater than the temperature of electrons by more than six orders of magnitude. It is unlikely that such differences in the temperatures actually do realize in the nature [7]. Therefore, in the frequency domain concerned, the plasma can be considered as a purely electronic, i.e. the role of the ions is reduced only to neutralize the charge of electrons. The dispersion relation (26) has to imaginary terms. The first one describes collisionless Cherenkov absorption of the plasma waves. Whereas, the second one has purely collisional nature and describes dissipation of the fields energy in via collisions (electronic friction) [7]. The difference induced by the presence of the beam of neutral particles is presented by a factor (see, Ref. [7] for comparison)

$$\left[1-\frac{\vec{k}\cdot\vec{V}_0}{\omega}\right]. \tag{27}$$

In addition to the high frequency longitudinal oscillations in isotropic collsionless plasma there also exist low frequency oscillations, so called, Ion-acoustic waves. They exist in highly non-isotermal plasma, where $T_e \gg T_i$. Phase velocity of these waves lies in the $V_{Ti} \ll \omega/k \ll V_{Te}$ domain. It is obvious, that such waves should also exist in collisional plasma if the collisions are sufficiently rare. Thus, when $\omega \ll \nu_{in}$ and $|\omega + i\nu_{en}| \ll kV_{Te}$ in the $V_{Ti} \ll \omega/k \ll V_{Te}$ phase velocity domain we obtain following dispersion relation

$$\varepsilon^l = 1 + \left[\frac{\omega_{Le}^2}{k^2 V_{Te}^2}\left(1 + i\sqrt{\frac{\pi}{2}}\frac{\omega}{kV_{Te}}\right) - \right.$$

$$\left.\frac{\omega_{Li}^2}{\omega^2}\left(1 + \frac{3k^2V_{Ti}^2}{\omega^2}\right) + i\left\{\sqrt{\frac{\pi}{2}}\frac{\omega\omega_{Li}^2}{k^3V_{Ti}^3}\exp\left[-\frac{\omega^2}{2k^2V_{Ti}^2}\right] + \frac{\omega_{Li}^2\nu_{in}}{\omega^3}\right\}\right]\left[1 - \frac{\vec{k}\cdot\vec{V}_0}{\omega}\right] = 0. \tag{28}$$

In the latter equation we have used also following asymptotic expansion

$$\bar{\phi}(z) = 2z^2 - \frac{4}{3}z^4 + ... - i\sqrt{\pi}ze^{-z^2}, \quad \text{when} \quad |z| \ll 1 \tag{29}$$

to the first order.

Again, we note that the difference induced by the presence of the beam of neutral particles is presented by a factor given by factor Eq.(27) (see, Ref. [7] for comparison).

This concludes presentaion of the kinetic model of three component, weakly ionized, collisional plasma with a beam of neutral particles. We have generalized the results of Ref. [7] by allowing neutral particles to have regular velocity (i.e. by allowing for the existence of a beam of neutrals). We have shown that the novel, generalized dispersion relations for linear perturbations obtained in this paper differ substantially from those of Ref. [7]. Finally, we would like to conclude outlining, once again, the possible relevance of our model for the better understanding of the plasma heating process by a neutral beam injection.

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[1] M. Murakami, R. C. Isler, J. F. Lyon et al., Phys. Rev. Lett. **39**, 615 (1977).
[2] H. Eubank, R. Goldston, V. Arunasalam et al., Phys. Rev. Lett. **43**, 270 (1979).
[3] J.D. Callen and J.A.Rome, Nuclear Fusion **20**, 501 (1980).
[4] M. Olsson, P. Vanbelle, S. Conroy, T. Elevant, and G. Sadler, Plasma Phys. Control. Fusion **35**, 179 (1993).
[5] G. Taylor, B. Leblanc, M. Murakami et al., Plasma Phys. Control. Fusion **38**, 723 (1996).
[6] P. G. Carolan, N. J. Conway, M. R. Tournianski, M. P. S. Nightingale, and M.J. Walsh, Plasma Phys. Rep. **24**, 206, (1998).
[7] A.F. Alexandrov, L.S. Bogdankevich, A.A. Rukhadze (1988), *Foundations of Plasma Electrodynamics*, Vis'shaya Shkola Press, Moscow (in Russian, English translation exists).
[8] D.B. Melrose (1986), *Instabilities in Space and Laboratory Plasmas*, Cambridge University Press, Cambridge.